A multiple ship routing and speed optimization problem under time, cost and environmental objectives

Wen, M.; Pacino, Dario; Kontovas, C.A.

Published in:
Transportation Research. Part D: Transport & Environment

Link to article, DOI:
10.1016/j.trd.2017.03.009

Publication date:
2017

Document Version
Peer reviewed version

A multiple ship routing and speed optimization problem under time, cost and environmental objectives

M. Wen[a,*], D. Pacino[b], C. A. Kontovas[c], H. N. Psaraftis[b]

[a] *Department of Mathematical Science, Xi'an Jiaotong-Liverpool University, 111 Ren Ai Road, Suzhou, Jiangsu, 215123, China*
[b] *Department of Management Engineering, Technical University of Denmark, 2800 Kgs. Lyngby, Denmark*
[c] *Department of Maritime and Mechanical Engineering, Liverpool John Moores University, L3 3AF Liverpool, United Kingdom*

Abstract

The purpose of this paper is to investigate a multiple ship routing and speed optimization problem under time, cost and environmental objectives. A branch and price algorithm as well as a constraint programming model are developed that consider (a) fuel consumption as a function of payload, (b) fuel price as an explicit input, (c) freight rate as an input, and (d) in-transit cargo inventory costs. The alternative objective functions are minimum total trip duration, minimum total cost and minimum emissions. Computational experience with the algorithm is reported on a variety of scenarios.

Keywords: Ship speed optimization, multi-commodity pickup and delivery, Branch-and-Price, combined ship speed and routing

1. Introduction

Ships travel slower than the other transportation modes. As long-distance trips may typically last one to two months, the benefits of a higher ship speed mainly entail the economic added value of faster delivery of goods, lower inventory costs and increased trade throughput per unit time. However, fast ship

*Corresponding author
Email addresses: min.wen@xjtlu.edu.cn (M. Wen), darpa@dtu.dk (D. Pacino), c.kontovas@ljmu.ac.uk (C. A. Kontovas), hnpsar@dtu.dk (H. N. Psaraftis)

Preprint submitted to Transportation Research Part D: Transport and Environment March 28, 2017

speeds entail increased emissions as the latter are proportional to fuel burned, which is an increasing function of ship speed. At the same time, the above benefits may become elusive whenever shipping markets are depressed and whenever fuel prices are on the increase. In such situations, ships tend to slow down, and slow steaming is a prevalent practice.

Because of the non-linear relationship between ship speed and fuel consumption, a ship that goes slower will burn much less fuel and produce much fewer emissions than the same ship going faster. Hence speed reduction is a tool that could reduce both fuel costs and emissions at the same time, and may potentially constitute a win-win proposition. It is certainly a prime tool for improving a ship's environmental performance, provided of course the relevant opportunity is adequately exploited.

In the charter (tramp) market, those who pay for the fuel, that is, the ship owner whose ship trades on the spot market, or the charterer if the ship is on time or bare-boat charter, will typically choose ship speed as a function of two main input parameters: (i) the fuel price and (ii) the market freight rate. In periods of depressed market conditions, as is the typical situation in recent years, ships tend to slow steam. The same is the case if bunker prices are high. Conversely, in boom periods or in case fuel prices are low, ships tend to sail faster.

A similar situation plays out in the liner market. Container and Ro-Ro operators typically operate a mixed fleet of vessels, some of which are owned vessels and some are chartered from independent owners who are not engaged in liner logistics. In either case, fuel is paid for by the liner operator. The operator receives income from the multitude of shippers whose cargoes are carried on the ship and the rates charged to these shippers can be high or low depending on the state of the market. As in the charter market, high fuel prices and/or depressed market conditions imply lower speeds for the fleet.

Investigating the economic and environmental implications of ship speed is not new in the maritime transportation literature and this body of knowledge is rapidly growing. In [1], some 42 relevant papers were reviewed and a taxonomy of these papers according to various criteria was developed. More papers dealing with ship speed are being published, as documented by the above paper's Google Scholar citations, which in October 2016 stood at 110, more than double the number a year before. Last but not least, a limited number of papers in recent years consider combined ship routing and speed decision problems. It is fair to say that this particular research area is still a new one, and much potential for further development still exists.

In that context, the purpose of this paper is to investigate a multiple ship routing problem with simultaneous speed optimization and under alternative objective functions. A heuristic branch-and-price algorithm as well as a constraint programming model are developed that consider (a) fuel consumption as a function of payload, (b) fuel price as an explicit input, (c) freight rate as an input, and (d) in-transit cargo inventory costs. The alternative objective functions are minimum total trip duration, minimum total cost and minimum emissions. Computational experience with the algorithm is reported on a variety of scenarios. Moreover, in order to evaluate the quality of the heuristic, an exact constraint programming model has also been developed. The reason for not comparing with an exact version of the branch-and-price algorithm is that the pricing problem is non-linear and that no known methods are available for solving it to optimality. This made constraint programming a natural choice.

We clarify right at the outset that weather routing considerations are outside the scope of this paper. Weather routing involves choosing the ships path and speed profile between two specified ports under variable and dynamically changing weather conditions. In weather routing, the ships fuel consumption function depends not only on ship speed and payload, but also on the prevailing weather conditions along the ships route, including wave height, wave direction, wind speed, wind direction, sea currents, and possibly others. Weather routing models (see for instance [2], among many others) take these factors into account. But models in a ship routing and scheduling context, including those developed in our paper, take a simpler approach: they do not deal with the problem of determining the best path between two ports, and they implicitly factor the average weather conditions the ship expects along its route into the fuel consumption function.

A related issue that we do not consider in this paper is the integration of risk and ship load monitoring data in the decision making process for optimal ship routing. Related research considers the impact of weather variables on ship safety attributes along a ships route. These include a ships structural integrity, the safety of the passengers, and possibly others. For an exposition see [3].

The rest of this paper is organized as follows. Section 2 discusses how some problem parameters that are considered important are treated in the literature. Section 3 describes the problem and Section 4 develops two mathematical formulations for it, a set partitioning formulation and a compact formulation. Section 5 develops a heuristic Branch-&-Price algorithm for the problem, together with an alternative constraint programming approach for comparison purposes. Section 6 describes and interprets the computational results and finally Section 7 presents the conclusions of the paper.

2. Which problem parameters are important? A focused look at the literature

It is outside the scope of this paper to conduct yet another full review of the literature, that close to the previous one. Rather, we list a number of input parameters and model assumptions that we consider important in ship speed optimization, and observe how these parameters are treated in a limited sample of the literature. In that context, the following may or may not be true in a model in which ship speed is a decision variable:

(a) fuel consumption is a function of payload,

(b) fuel price is an input (explicit or implicit),

(c) freight rate is an input, and

(d) in-transit cargo inventory costs are considered.

All of the above (a) to (d) can be important. The degree of importance depends on the particular scenario examined. Briefly below we argue about the importance of each.

As regards (a), it is clear that ship payload can drastically influence fuel consumption (and hence emissions) at a given speed, with differences of the order 30% between fully laden and ballast conditions being observed for the same speed. The dependency on payload is more prevalent in tankers and bulk carriers that sail either full or empty and less prevalent in other types of ships, which can be partially laden (container ships) or their payload does not change much (Ro-Ro ships, passenger ships, cruise ships). The functional relationship between ship speed and payload on the one hand and fuel consumption on the other is typically non-linear and may not even be available in closed form. Section 3 presents a realistic closed-form approximation.

As regards (b) and (c), in [1] it was shown that it is mainly the non-dimensional ratio of fuel price over the market spot rate that determines optimal ship speed, with higher speeds corresponding to lower such ratios. Optimal here is defined as maximizing the average per day profit of the ship owner. This reflects the typical behavior of shipping companies, which tend to slow steam in periods of depressed market conditions and/or high fuel prices and go faster if the opposite is the case. As regards (b), fuel price may be given either explicitly in the model, in the form of a distinct input, or implicitly, whenever a fuel cost function is given. An implicit formulation has the drawback of not allowing someone to directly analyze the functional dependency between fuel price and optimal speed.

Finally as regards (d), in-transit inventory costs accrue while the ship is in transit, and they can be a non-trivial component of the cost that the owner of the cargo (that is, the charterer) bears if the ship will sail at a reduced speed. They can be important if timely delivery of the cargo is significant. They can also be important if the voyage time and/or the quantities to be transported are non-trivial. This can be the case in long-haul problems. In-transit inventory costs are also important for the ship owner, as a charterer will prefer a ship that delivers his cargo earlier than another ship that sails slower. Thus, if the owner of the slower ship would like to attract that cargo, he may have to rebate to the charterer the loss due to delayed delivery of cargo. In that sense, the in-transit inventory cost is very much relevant in the ship owner's profit equation, as much as it is relevant in the charterer's cost equation.

Table 1 lists a limited sample of papers and lists whether or not each of (a) to (d) above is true. Based on the table, we can advance the conjecture that whatever the shipping market and logistical context, ours is the only paper in the maritime literature that addresses a multiple ship scenario in which all of parameters (a) to (d) above are true.

Papers	Shipping market	Logistical context	Number of ships	(a) Fuel/payload	(b) Fuel price	(c) Freight rate	(d) In-transit cargo costs
[4]	Tramp	Fixed route	One	No	Explicit	Yes	No
[5]	Container	Fleet deployment	Many	No	Explicit	Yes	No
[6]	Tanker	World oil network	Many	Only for laden and ballast conditions	Explicit	No. Equilibrium spot rate computed	Yes
[7]	Container	Fixed route	Many	No	Explicit	No	No
[8]	Tramp	Pickup and delivery	Many	No	Implicit	No	No
[9]	Container	Fixed route	Many	No	Explicit	No	Yes
[10]	Tanker	Fixed route	Many	Only for laden and ballast conditions	Explicit	Yes	Yes
[11]	General	Fixed route	One	No	Implicit	No	No
[12]	Tramp	Pickup and delivery	Many	No	Implicit	For spot cargoes	No
[13]	General	Fixed or flexible route	One	For any loading condition	Explicit	Yes	Yes
[14]	Container	Fixed route in SECAs	Many	No	Explicit	No	No
[15]	Ro-Ro	Fleet deployment	Many	Only for laden and ballast conditions	Implicit	No	No
[16]	Ro-Ro	Route selection in SECAs	One	No	Explicit	No	No
[17]	Container	Disruption management	One	No	Implicit	No	No
[18]	Container	Fleet deployment	Many	For any loading condition	Explicit	Yes	No
[19]	Container	Berth allocation, virtual arrival	Many	No	Implicit	No	No
[20]	General	Speed optimization in a dynamic setting	One	No	Explicit	Yes	No
This Paper	General	Pickup and delivery	Many	For any loading condition	Explicit	Yes	Yes

Table 1: Sample of speed papers and whether parameters (a) to (d) are included in the model. The parameters indicate: (a) If fuel consumption is a function of payload, (b) if fuel price is an implicit or explicit input, (c) is freight rate is an input, (d) if in-transit cargo inventory costs are considered.

It should be clarified here that no time windows are assumed in our model. Whereas this may be perceived as a potential limitation, there is a specific reason that we do not consider them: time windows may implicitly or explicitly dictate what the speed of the ship might be (at least in some trip legs) and, as such, may limit the flexibility of choosing an optimal speed according to a prescribed objective. They would also prevent one to see the variety of solutions under alternative objectives, since if speed is more or less fixed, some of the problem's objectives may be rendered to produce the same solutions. It should also be noted that in practice time windows are not really exogenous inputs, as most of the literature assumes, being usually the subject of negotiation and agreement between the shipper and the shipping company so that feasible solutions are obtained. It is also important to consider the fact that in-transit cargo inventory costs will make sure that cargo is delivered on time and not delayed, which makes this objective component a surrogate for time-windows.

3. Problem description and mathematical formulation

We consider the optimization of routes and speeds of an heterogeneous fleet that needs to pickup and deliver a set of cargoes. Each cargo has a specific weight, pickup and delivery destination. Cargoes cannot be split and should be picked up by exactly one ship during one visit, however the ships are allowed to make multiple visits in a ports if this is necessary.

We assume that the ships used for the delivery are on time charter with given freight rates (expressed in $/day). These freight rates are assumed to be known for each ship and independent of charter duration[1]. In general they will be different for each ship, as they depend on ship size. Each ship is initially located at a given port and has a known payload capacity that cannot be exceeded. A ship can sail at different speeds on different legs of the route as long as the speeds are within its feasible speed range (which is dictated by the ship's engine size and technology).

[1] In general the time charter rate is a function of charter duration, but for charters of the same time range (e.g. short term as opposed to long term) one can assume that the rate is independent of charter duration.

The daily fuel consumption of each ship (in tons/day) is given by a function $f(v, w)$ of the ship's speed v (in nautical miles/day, or knots) and payload w (in tons). In this work, we use the realistic closed-form approximation of f given in [13]:

$$f(v, w) = G(P + v^T)(w + A)^{2/3} \tag{1}$$

where $G > 0$, $P \geq 0$ and $T \geq 3$ are ship related constants, and A is the modified 'lightship weight', that is, the weight of the ship if empty including fuel and other consumables but without any cargo on board. Strictly speaking, f must take into account the reduction in the ship's total displacement due to fuel being consumed along the ship's route. However, since displacement would not change much as a result of that consumption, one can practically assume f independent of en-route fuel consumption. In addition, we consider a heterogeneous fleet, meaning that the initial ports, the capacities, the freight rates, the feasible speed ranges, and the fuel consumption parameters can be different for each ship.

Equation (1) assumes that the average weather conditions that the ship expects along its route are implicitly factored into the fuel consumption function. As stated earlier, and as this is not a weather routing model, no explicit consideration of weather variables is included.

We assume that the charterer (the cargo owner) bears all cargo inventory costs. These have two components: 1) *port inventory cost*, the cost due to cargo waiting to be picked up, and 2) *in-transit inventory cost*, the cost due to cargo being in transit. These inventory costs are assumed to be linear in time and in cargo volume. A zero port inventory cost assumes that the cargoes are available at the origin ports in a 'just-in-time' fashion.

The objective of this problem is to minimize the total cost over all route legs. Three cost components are considered: fuel costs, cargo inventory costs and time charter costs.

As pointed out in [13], for a single ship and a given route, the total cost of an individual route leg (L, L') is equal to

$$COST(L, L') = \left(UG(P + v^T)(w + A)^{2/3} + \alpha u + \beta w + F \right) \cdot \frac{d_{LL'}}{v} \qquad (2)$$

where

$d_{LL'}$: the distance of leg (L, L') (in nautical miles)

U: the fuel price (in \$/ton)

F: the time charter freight rate of the ship (in \$/day)

α: the unit cargo port inventory cost (in \$/tons/day)

β: the unit cargo in-transit inventory cost (in \$/tons/day)

u: the amount of cargo still waiting to be picked up (in tons)

It is obvious that $COST(L, L')$ is a function of speed v when the route sequence is fixed. To obtain the speed that leads to a minimum value of $COST(L, L')$, we just need to identify the speed that minimizes (1) and compare it with the ship's speed range $[v_{LB}, v_{UB}]$. This speed point can be obtained by setting the first derivative of $COST(L, L')$ to zero as follows:

$$\hat{v} = \left(\frac{UGP(w + A)^{2/3} + \alpha u + \beta w + F}{UG(w + A)^{2/3}(T - 1)} \right)^{\frac{1}{T}} \qquad (3)$$

The optimal speed v^* should be $\hat{v}$ if $v_{LB} \leq \hat{v} \leq v_{UB}$, v_{LB} if $\hat{v} \leq v_{LB}$, and v_{UB} if $\hat{v} \geq v_{UB}$.

3.1. Mathematical Formulations

We can define a problem with n cargoes and m ships on a graph $G = (N, E)$, where N is the set of all the nodes and E is the set of feasible arcs in the graph. Let $P = \{1, ..., n\}$ denote the set of pickup nodes and $D = \{n+1, ..., 2n\}$ the set of delivery nodes. Cargo i is represented by the node pair $(i, n+i)$. Let K denote the set of ships. Ship $k \in K$ starts from node $o(k)$ and returns to a dummy node $d(k)$. Let d_{ij} denote the distance between node i and node j. If the ships are not required to end their journey at specific ports, we can just set $d_{id(k)} = 0$ for all i and k. The set of all the nodes is $N = P \cup D \cup \{o(1), ..., o(m)\} \cup \{d(1), ..., d(m)\}$.

Let $N_i^+ = \{j : (i,j) \in E\}$ and $N_i^- = \{j : (j,i) \in E\}$ be the set of nodes that can be reached from node i, and can reach node i respectively.

For each node i, let H_i denote the amount of cargo to be loaded, $H_i > 0$ for $i \in P$, and $H_i = -H_{i-n}$ for $i \in D$. The per unit volume and per unit time cargo port inventory cost α and cargo in-transit inventory cost β are assumed the same for all the cargoes. Each ship $k \in K$ has a capacity Q_k and can sail at any speed between its minimum speed L_k and maximum speed U_k. The freight rate of ship k is F_k per unit time. Let A_k denote ship k's lightship weight. Let G_k, P_k and T_k denote the corresponding parameters in the fuel consumption formula (1) for ship k. The per unit volume fuel cost is denoted by U.

3.1.1. A compact formulation

Let the binary decision variable x_{ij}^k be 1 if ship $k \in K$ sails from node $i \in N$ to $j \in N$ and 0 otherwise. Let auxiliary variable $\hat{v}_{ij}^k$ denote the optimal speed from (3) for ship k on leg (i,j), and let the decision variable v_{ij}^k be the actual sailing speed of ship k when sailing from node i to j. The variable q_i^k represents the load of ship k after loading/unloading cargo at node i. For the purpose of evaluating the total cost of ship k on leg (i,j), we need to keep track on the total weight of cargo not yet picked up while ship sails on each leg. We therefore define variable t^k as the total weight ship k delivers on the entire route, and variable h_i^k as the total weight ship k has already delivered after loading/unloading at node i. The total weight of the cargo waiting to be picked up by ship k after visiting node i is $t^k - h_i^k$. Finally, let u_i be the sequence variable used to eliminate subtours.

$$z^* = \min \sum_{k \in K} \sum_{(i,j) \in E} x_{ij}^k \left(UG_k(P_k + {v_{ij}^k}^{T_k})(q_i^k + A_k)^{2/3} + \alpha(t^k - h_i^k) + \beta q_i^k + F_k \right) \frac{d_{ij}}{v_{ij}^k} \quad (4)$$

$$\text{s.t.} \sum_{k \in K} \sum_{j \in N_i^+} x_{ij}^k = 1 \quad \forall i \in P \quad (5)$$

$$\sum_{j \in N_{o(k)}^+} x_{o(k)j}^k = 1 \quad \forall k \in K \quad (6)$$

$$\sum_{j \in N_i^+} x_{ij}^k - \sum_{j \in N_i^-} x_{ji}^k = 0 \quad \forall i \in P \cup D, k \in K \quad (7)$$

$$\sum_{j \in N_{d(k)}^-} x_{jd(k)}^k = 1 \quad \forall k \in K \quad (8)$$

$$u_j \geq u_i + 1 - M(1 - x_{ij}^k) \quad \forall (i,j) \in E, k \in K \quad (9)$$

$$\sum_{j\in N_i^+} x_{ij}^k - \sum_{j\in N_{n+i}^+} x_{n+i,j}^k = 0 \quad \forall i \in P, k \in K \tag{10}$$

$$u_{n+i} \geq u_i \quad \forall i \in P \tag{11}$$

$$t^k = \sum_{j\in N_i^+} \sum_{i\in P} H_i x_{ij}^k \quad \forall k \in K \tag{12}$$

$$q_j^k \geq q_i^k + H_i x_{ij}^k - M(1 - x_{ij}^k) \quad \forall (i,j) \in E, k \in K \tag{13}$$

$$h_j^k \geq h_i^k + \max\{0, H_i\} x_{ij}^k - M(1 - x_{ij}^k) \quad \forall (i,j) \in E, k \in K \tag{14}$$

$$\max\{0, H_i\} \leq q_i^k \leq Q_k \quad \forall i \in N, k \in K \tag{15}$$

$$\hat{v}_{ij}^k = \left(\frac{UG_k P_k (q_i^k + A_k)^{2/3} + \alpha(t^k - h_i^k) + \beta q_i^k + F_k}{UG_k (q_i^k + A_k)^{2/3} (T_k - 1)} \right)^{\frac{1}{T_k}} \quad \forall (i,j) \in E, k \in K \tag{16}$$

$$L_k + \max\{0, \hat{v}_{ij}^k - L_k\} \cdot M \geq v_{ij}^k \geq L_k \quad \forall (i,j) \in E, k \in K \tag{17}$$

$$U_k \geq v_{ij}^k \geq U_k + \min\{0, \hat{v}_{ij}^k - U_k\} \cdot M \quad \forall (i,j) \in E, k \in K \tag{18}$$

$$\begin{aligned} \hat{v}_{ij}^k + \max\{0, L_k - \hat{v}_{ij}^k, \hat{v}_{ij}^k - U_k\} \cdot M \geq v_{ij}^k \geq \hat{v}_{ij}^k - \max\{0, L_k - \hat{v}_{ij}^k, \hat{v}_{ij}^k - U_k\} \cdot M \\ \forall (i,j) \in E, k \in K \end{aligned} \tag{19}$$

$$x_{ij}^k \in \{0,1\} \quad \forall (i,j) \in E, k \in K \tag{20}$$

$$t^k, h_i^k, q_i^k, \hat{v}_{ij}^k, v_{ij}^k \geq 0 \quad \forall i \in N, k \in K \tag{21}$$

$$u_i \in \mathbb{Z}_+ \quad \forall i \in N \tag{22}$$

The objective (4) minimizes the total cost of all the route legs. Constraints (5) make sure that each cargo is delivered by exactly one ship. Constraints (6)–(8) are the flow conversation constraints. Constraints (9) eliminate the sub-tours. Constraints (10) and (11) are so-called paring constraints and precedence constraints that enforce each cargo to be first picked up and then delivered by the same ship. Constraints (12) calculate the total weight of cargoes assigned to each ship. Constraints (13) and (14) keep track on the load of the ship and the total weight the ship has already delivered after loading/unloading at a node. Constraints (15) are the ship capacity constraints. Constraints (16) calculates the $\hat{v}_{ij}^k$ value for ship k on leg (i,j) in the same way as (3). The optimal speed v_{ij}^k is determined by constraints (17)–(19). Finally, the decision variables are defined by (20)–(22).

3.1.2. A Set Partitioning formulation

This problem can also be formulated as a Set Partitioning Problem. Let R^k be the set of feasible routes for ship $k \in K$, all of which start from node $o(k)$, end at node $d(k)$, satisfy the paring and precedence constraints, and are feasible with respect to the ship's capacity and speed range. Let c_r^k denote the cost of route $r \in R^k$ for ship k, calculated as the sum of total cost over all the legs in the route. Parameter a_{ir} equals 1 if route r covers cargo i, and 0 otherwise. Let the binary variable y_r^k be 1 if route $r \in R^k$ is taken by ship k, and 0 otherwise. The problem can then be formulated as follows:

$$z^* = \min \sum_{k \in K} \sum_{r \in R^k} c_r^k y_r^k \tag{23}$$

$$\text{s.t.} \sum_{k \in K} \sum_{r \in R^k} a_{ir} y_r^k = 1 \qquad i \in P \tag{24}$$

$$\sum_{r \in R^k} y_r^k \leq 1 \qquad k \in K \tag{25}$$

$$y_r^k \in \{0,1\} \qquad \forall r \in R^k, k \in K \tag{26}$$

The objective is to minimize the cost of the selected routes in such way that each cargo is delivered (24) and each ship is assigned to at most one route (25).

The LP relaxation of the set partitioning formulation will always provide the same or better lower bound compared to the LP relaxation of the compact formulation.

4. Solution methods

We propose two solution methods: a Heuristic Branch-and-Price (H-B&P) in Section 4.1 and a Constraint Programming Model (CPM) in Section 4.2.

4.1. Heuristic Branch-and-Price

Solving model (23)–(26) directly by an IP solver requires the enumeration of all feasible ship routes, which seems impossible given the huge size of feasible routes. Instead, we solve the model by a heuristic branch-and-price algorithm similar to [21]. Branch-and-Price (B&P) is a version of branch-and-bound, where the linear programming (LP) relaxation at each node of the branch-and-bound tree is obtained by using the Column Generation (CG) method ([22]). The LP relaxation of the problem (denoted by LP-SP) can be obtained by relaxing the binary constraints (26) as follows:

$$y_r^k \geq 0 \qquad\qquad \forall r \in R^k, k \in K$$

The CG starts by solving a restricted LP-SP, called the *master problem*, where only a subset of ship routes are considered, and then gradually generates the rest of the routes that can potentially improve the objective function and adds them to the model. A solution to the master problem provides the the dual variables π_i and λ^k corresponding to constraints (24) and (25). These values can be used to calculate the reduced cost of a route $r \in R^k$ for ship $k \in K$ as $\hat{c}_r^k = c_r^k - \sum_{i \in P} a_{ir}\pi_i - \lambda^k$. From the theory of the Simplex method, adding a route with negative reduced cost can possibly produce an improved LP solution. If $\hat{c}_r^k \geq 0$ for all feasible route r and all ship k then the solution to the restricted LP-SP is also optimal to the full LP-SP. Otherwise, the route with negative reduced cost should be added to the master problem and the master problem needs to be solved again to get new dual variables.

Finding the route with the lowest $\hat{c}_r^k$ is done by solving a *pricing problem*. In our case, the pricing problem is an elementary shortest path problem with capacity, pickup and delivery, variable speed and variable arc costs, in which the speed and cost of each arc varies as the route sequence varies. Here we examine how to define the speed and arc cost in the shortest path problem related to ship $k \in K$. For a given route $r \in R^k$, the speed of leg (i, j) in route r is defined as

$$v_{ijr}^k = \begin{cases} L_k & \text{if } \hat{v}_{ijr}^k \leq L_k \\ \hat{v}_{ijr}^k & \text{if } L_k \leq \hat{v}_{ijr}^k \leq U_k \\ U_k & \text{if } U_k \leq \hat{v}_{ijr}^k \end{cases}$$

where

$$\hat{v}_{ijr}^{k} = \left(\frac{UG_k P_k (w_{ijr} + A_k)^{2/3} + \alpha u_{ijr} + \beta w_{ijr} + F_k}{UG_k (w_{ijr} + A_k)^{2/3} (T_k - 1)} \right)^{\frac{1}{T_k}}$$

and w_{ijr} and u_{ijr} are the payload and the weight to be picked up during leg (i, j) in route r. The cost of leg (i, j) in a route r in the pricing problem is calculated as

$$\hat{c}_{ijr}^{k} = \begin{cases} c_{ijr}^{k} - \pi_i & \text{if } i \in P \\ c_{ijr}^{k} & \text{if } i \in D \\ c_{ijr}^{k} - \lambda^k & \text{if } i = o(k) \end{cases}$$

where

$$c_{ijr}^{k} = \left(UG_k \big(P_k + (\hat{v}_{ijr}^{k})^{T_k} \big) (w_{ijr} + A_k)^{2/3} + \alpha u_{ijr} + \beta w_{ijr} + F_k \right) \frac{d_{ij}}{\hat{v}_{ijr}^{k}}.$$

By using the above defined arc cost $\hat{c}_{ijr}^{k}$, the cost of route r will equal the reduced cost of the corresponding variable.

The resource constrained shortest path problem is usually solved by labeling algorithms [23]. However, solving our pricing problem to optimality can be time consuming given its high complexity. To be able to solve the problem in reasonable computational time, we use a cheapest insertion heuristic. The heuristic starts from a route containing only one cargo, and gradually inserts the remaining cargoes that least increases the reduced cost of the route. During the insertion, we keep track of the routes with most negative reduced costs. The procedure is repeated with every cargo as a starting point and for every ship $k \in K$. If the heuristic fails to find any route with negative reduced cost, the column generation procedure stops and proceeds as if we have solved the LP-SP to optimality. However, we can not guarantee the optimality due to the fact that the pricing problem is solved heuristically. We call this method of solving the LP-SP as heuristic column generation (H-CG).

If the solution obtained by the H-CG is an integer solution, the H-B&P algorithm stops. Otherwise, we branch on the arc variables as suggested in [24]. The algorithm uses strong branching in order to decide which arc to branch on.

A number, γ, of branching candidates are evaluated by enforcing the branch and computing the resultant improvement in the lower bounds (Δ_1 and Δ_2) in the two child nodes. Following [25], the algorithm chooses the branch that maximizes

$$\mu \min\{\Delta_1, \Delta_2\} + (1 - \mu) \max\{\Delta_1, \Delta_2\}$$

where $0 \leq \mu \leq 1$ is a parameter.

The H-B&P stops until all the nodes in the search tree are explored. Since the LP-SP is solved by the H-CG and the solution found by the H-B&P is not necessarily optimal, it can potentially be improved. In a post-optimization phase, we use an IP solver to solve the set partitioning model with all the columns found in the branch-and-price procedure. The solution to such model is at least as good as the solution found by the branch-and-price.

4.2. A Constraint Programming model

Changing the solution method of the pricing problem with an exact approach, could give use the possibility of comparing our heuristic solutions to the optimal ones. In the literature, the only know method to solve a similar problem is the dynamic programming approach proposed in [13]. This procedure is, however, not able to scale to multiple vessels and a larger set of ports. Thus, we sought an alternative solution approach, constraint programming, which not only it is an exact method but it can also deal with non-linear functions.

Constraint programming is a search based approach to solve constraint satisfaction problems. Problems are modeled in terms of variables and their domains, and a set of constraints (relations between variables). At each step of the search, specialized filtering algorithms analyze the constraints and remove infeasible values from the variables domain. In case of an optimization problem, the search can be performed within a branch & bound algorithm which thus allows the finding of optimal solutions. The filtering and search algorithms are often part of a solver (as it is in this case). We thus only present a description of the model and refer the reader to [26] for further information.

The model is an adaptation of the VRPPD model presented in [26] and uses the same notation and node representation described in Section 3.1. A solution to the problem is represented by a sequence of nodes determined by the variable $p_i \in N$, which indicates the node immediately before node $i \in N$. The speed used to reach node i from its preceding node p_i is decided by the variable $v_i \in \mathbb{R}_+$. Furthermore, the model makes use of a number of auxiliary variables: $l_i \in \mathbb{Z}_+$ is the load of the ship going to node i, $s_i \in K$ is the ship sailing to node i, $r_i \in \mathbb{Z}_+$ is the amount of cargo yet to be picked-up after leaving node i, and $c_i \in \mathbb{R}_+$ is the total cost at node i. Finally, a number of variables have been introduced to ease the modeling of the problem: $o_i \in N$ is the node at position i in the solution sequence (e.g. if node 5 is the first in the sequence then it must be the case that $o_1 = 5$), $b_i \in N$ is the position of node i in the sequence (e.g. if node 5 is the first in the sequence then it must be the case that $b_5 = 1$), and $a_{ij} \in \{0,1\}$ which is 1 iff node i is visited after node j and 0 otherwise.

$$circuit(\mathcal{P}, \mathcal{D}) \tag{27}$$

$$p_{o(k+1)} = d(k) \quad \forall k \in K \tag{28}$$

$$s_{o(k)} = k \quad \forall k \in K \tag{29}$$

$$s_{d(k)} = k \quad \forall k \in K \tag{30}$$

$$s_{p_i} = s_i \quad \forall i \in P \cup D \tag{31}$$

$$l_i = l_{p(i)} + H_i \quad \forall i \in N \tag{32}$$

$$l_i \leq Q_{s_i} \quad \forall i \in N \tag{33}$$

$$o_i \leq o_{n+i} \quad \forall i \in P \tag{34}$$

$$o_i = p_{o_{i+1}} \quad \forall i \in N \tag{35}$$

$$allDifferent(\mathcal{O}) \tag{36}$$

$$s_i = s_{n+i} \quad \forall i \in P \tag{37}$$

$$L_{s_i} \leq v_i \leq U_{v_i} \quad \forall i \in N \tag{38}$$

$$optimalSpeed(v_i, l_i, s_i, r_i) \quad \forall i \in N \tag{39}$$

$$o_i = j \Leftrightarrow b_j = i \quad \forall i, j \in N \tag{40}$$

$$a_{ij} = (b_i < b_j) \wedge (v_i = v_j) \quad \forall i, j \in N \tag{41}$$

$$r_i = \sum_{j \in P} d_j a_{ij} \quad \forall i, j \in N \tag{42}$$

$$costFunc(c_i, v_i, l_i, s_i, r_i) \quad \forall i \in N \tag{43}$$

Constraint (27) uses the global constraint *Circuit* [26] to force the set $\mathcal{P} = \{p_i : i \in N\}$ of all p_i variables to form an Hamiltonian circuit. Moreover, this constraint keeps track of the sailed distance at each node, where $\mathcal{D}$ is the distance matrix. The filtering algorithm also imposes sub-tours elimination. Constraints (28) - (31) are related to the vessel. Constraint (28) forces the depot end node ($d(k)$) of vessel $k \in K$ to be immediately followed by the next vessel's depot start node ($o(k+1)$). This constraint not only ensures the consistency of the solution, it also removes symmetrical sequences where the routes of the different ships exchange position in the solution encoding. Constraint (29) - (30) binds the s_k ship variables to their corresponding depot start and end node. Constraint (31) imposes that only one ship can be present in one route. Note that it is possible to have multiple routes since the constraint is only posted for the the pickup (P) and delivery (D) nodes. The cargo and ship capacity are constrained by (32) and (33). The first ensures that the load of the ship visiting node $i \in N$ (l_i) is updated by the demand H_i, while the second ensures that the capacity of the assigned ship is not exceeded. Constraint (34) forces a precedence between a pickup node $i \in P$ and its corresponding delivery node $n+i$. The order variables o_i are linked to the predecessor variables p_i via constraint (35). To improve pruning, an *allDifferent* constraint [26][2] is imposed over the set of order variables ($\mathcal{O} = \{o_i : i \in K\}$) in constraint (36). Constraint (37) ensures that the same ship that picks up a cargo also delivers it. The speed at each node is limited to the minimum and maximum speed of

[2]Imposes that each variable in the given set must have a distinct value

the assigned ship by constraint (38). In order to model the speed of the ship we have, in Constraint (39), implemented a dedicated filtering algorithm, which, based on the optimal speed equation from [13], ensures bound consistency on the speed variables. In order to model the remaining cargo to be loaded (r_i) at a node, we used a binary variable a_{ij} indicating if node i is visited before node j and they are both in the same route (or equivalently if they are visited by the same ship). To do so we needed the dual version of the order variable o_i, which in Constraint (40) is obtained using a so called *channeling constraint*. Using the b_i variable, Constraint (41) can then define the a_{ij} variables. The remaining cargo load (r_i) is then obtained by collecting the demands yet to be visited (42). Another bound consistency filtering algorithm has been implemented for the cost calculation (43), which binds the different cost component to the cost variable c_i. The filtering algorithms used in (39) and (43) are explained in detail in Section 4.3.

The objective function (44) is then the minimization of the sum of all cost components c_i.

$$z^* = \min \quad \sum_{i \in N} c_i \tag{44}$$

4.3. Speed and cost filtering algorithms

The *optimalSpeed*() and *costFunc*() algorithms filter values respectively from the domain of the speed (v_i) and cost (c_i) variables. Both algorithm force the so called bound consistency, meaning that they can only adjust the lower and upper bound of the domains (contrary to arc-consistency where values within the domain set can be removed). Since both filtering algorithms have a dependency from other variables, which might have not yet been assigned, we must be able to work with the domain of these variable. For simplicity, let us define the lower bound of a variable x to be $\check{x}$ and the upper bound to be $\hat{x}$. Thus, from the variable $s_i \in K$, $\check{s}_i$ and $\hat{s}_i$ are respectively the smallest and largest, feasible, vessel index for node $i \in N$. Let G_i, P_i, T_i, F_i and A_i denote the corresponding parameters in Section 3.1 for a ship sailing to node $i \in N$. The per unit volume fuel cost is denoted by U. Again, for simplicity, we abuse the notation and define $\check{G}_i, \check{P}_i, \check{T}_i, \check{F}_i$ and $\check{A}_i$, to be the smallest values these coefficient can have at node $i \in N$, and $\hat{G}_i, \hat{P}_i, \hat{T}_i, \hat{F}_i$ and $\hat{A}_i$, to be the highest (e.g. $\hat{G}_i = \max_{j \in Dom(s_i)} G_j$ where $Dom(s_i)$ is the current domain of variable s_i for node $i \in N$).

For each $i \in N$ the *optimalSpeed*(v_i, l_i, s_i, r_i) filters the domain of the v_i variables as follows:

$$\hat{k}_1 = U\left(\hat{G}_i(\hat{l}_i + \hat{A}_i)^{\frac{2}{3}}\right) \tag{45}$$

$$\check{k}_1 = U\left(\check{G}_i(\check{l}_i + \check{A}_i)^{\frac{2}{3}}\right) \tag{46}$$

$$\hat{k}_2 = \hat{k}_1\hat{P}_i + \left(\alpha\hat{r}_i + \beta\hat{l}_i + \hat{F}_i\right) \tag{47}$$

$$\check{k}_2 = \check{k}_1\check{P}_i + \left(\alpha\check{r}_i + \beta\check{l}_i + \check{F}_i\right) \tag{48}$$

$$\hat{s}_i = \left(\frac{\hat{k}_2}{\check{k}_1(\check{T}_i - 1)}\right)^{\frac{1}{\check{T}_i}} \tag{49}$$

$$\check{s}_i = \left(\frac{\check{k}_2}{\hat{k}_1(\hat{T}_i - 1)}\right)^{\frac{1}{\hat{T}_i}} \tag{50}$$

Similarly, *costFunc*(c_i, v_i, l_i, r_i) filters the domain of the c_i variables as follows:

$$\hat{c}_i = \left[U\,\hat{G}_i(\hat{P}_i + \hat{v}_i^3)(\hat{l}_i + \hat{A}_i)^{\frac{2}{3}} + \alpha\hat{r}_i + \beta\hat{l}_i + \hat{F}_i\right]\frac{\hat{\delta}_i}{\check{v}_i} \tag{51}$$

$$\check{c}_i = \left[U\,\check{G}_i(\check{P}_i + \check{v}_i^3)(\check{l}_i + \check{A}_i)^{\frac{2}{3}} + \alpha\check{r}_i + \beta\check{l}_i + \check{F}_i\right]\frac{\check{\delta}_i}{\hat{v}_i} \tag{52}$$

where $\hat{\delta}_i$ and $\check{\delta}_i$ are respectively the longest and shortest distance to from the previous node in the sequence (e.g. $\hat{\delta}_i = \max_{j \in Dom(p_i)} d_{ij}$).

4.4. Search strategy

The model is solved using a dynamic branching that attempts at building routes backwards from each ship dummy end node. The strategy sequentially selects the first ship which route in not yet complete (which happens when one of the predecessor variable p_i is assigned to the dummy start node of the selected ship). It then attempts to assign the arc which incurs the highest cost (thus assigning a value to the p_i variables). Since the speed variables v_i are mainly derived by the rest of the variables, they are branched on at last. This branching is based on the traditional fail first strategy where the solver attempts at cutting as early as possible sub-optimal branches. The original strategy branches first on the variable with the smallest domain selecting a random value. During the experimental evaluation, the original strategy was able to provide faster optimal solutions to very small instances, but failed to provide even upper bound to larger ones.

5. Computational Results

This section presents the computational results of both solution methods on a set of generated realistic data. The H-B&P is implemented in C++ and run on a PC with Intel Core i7-3520M, 2.9Hz, 8GB RAM. The SP model in the H-B&P is solved by CPLEX 12.6. The parameters γ and μ in strong branching were set to $\frac{3}{4}$ and 15, as in [27] and [21]. The computational time is limited to 30 minutes. The CPM is implemented in C++ and uses Gecode 4.4 [28] and run on a similar Linux machine for 10 hours. In the following, Section 5.1 describes the testing data and Sections 5.2–5.4 present the results.

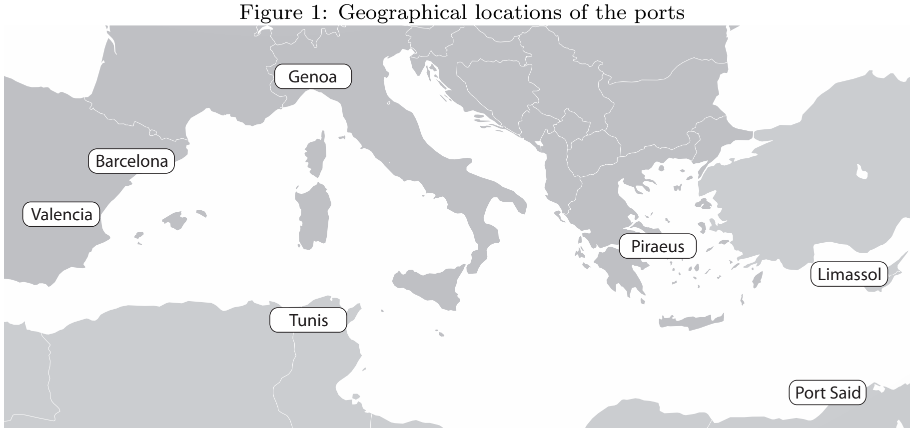
5.1. Data

Our instances contain cargoes that originate from 4-7 ports, whose geographical locations are illustrated in Figure 1. Distances between ports (in nautical miles) are taken from LinerLIB, a benchmark suite for liner shipping network design described in [29], and they are presented in Table 2.

The number and size of the cargoes for each instance group are randomly defined. Table 3 presents the number of cargoes and ports used in each group.

In each scenario there are up to 3 vessels that can be used, the size of which varies from small to large. These vessels are deployed in the Intra-Mediterranean container trade. Detailed ship characteristics such as ship's lightweight, total amount of cargo that can be transported (capacity), the range of sailing speeds,

Figure 1: Geographical locations of the ports



port ID (name)	1 (Tunis)	2 (Port Said)	3(Piraeus)	4(Genoa)	5(Valencia)	6(Barcelona)	7(Limassol)
1 (Tunis)	0	1192	701	472	560	492	1150
2 (Port Said)	1192	0	619	1446	1699	1620	228
3 (Piraeus)	701	619	0	906	1174	1095	554
4 (Genoa)	472	1446	906	0	512	356	1393
5 (Valencia)	560	1699	1174	512	0	165	1657
6 (Barcelona)	492	1620	1095	356	165	0	1562
7 (Limassol)	1150	228	554	1393	1657	1562	0

Table 2: Distance matrix (port distances in nautical miles)

the fuel consumption at the maximum speed as well as the freight rate (the per day price which a charterer pays a shipowner for the use of each ship) are presented in Table 4[3].

The fuel consumption per leg (for each ship) is calculated by using (1). In our instances we assume a cubic relationship between fuel consumption and speed, that is we set $P= 0$ and $T= 3$. By assuming the above, we are able to calculate the value of G that is in formula (1), such that at full capacity and at the maximum speed, the fuel consumption is equal to the "fuel consumption at

[3]The data of Table 4 are illustrative but realistic. They are drawn from various sources at the authors disposal, including private communication with industry contacts. The ships span the lower end of the containership size spectrum and we thought they would be a good example to test the models developed in the paper.

Instance group ID	G1	G2	G3	G4	G5	G6	G7	G8
# of cargoes	6	12	10	20	15	30	21	31
# of ports	4	4	5	5	6	6	7	7

Table 3: Instance data

Ship ID	1	2	3
Ship size	Small	Medium	Large
Freight rate ($/day)	6700	7800	10650
min speed (knots)	6	7	8
max speed (knots)	13	14	16
capacity (ton)	9400	11000	15000
Lightship weight (ton)	3500	5000	5000
fuel consumption at max speed (tons/day)	20	30	45

Table 4: Ship data

max speed" that is given in Table 4.

In order to estimate the bunker costs a base value of U equal to 300 $ per ton fuel is assumed.

As described in Section 3, the total inventory cost is also taken into account. Two types of inventory cost are assumed in this paper, in-transit inventory cost (β, which accrues from time cargo is on the ship until cargo is delivered) and port inventory cost (α, which accrues from time 0 until cargo is on the ship).

In the general case, we assume that β is related to cargo value. If the market price of the cargo at the destination (CIF price) is p $ per ton, then one day of delay in the delivery of one ton of this cargo will inflict a loss of $p \cdot r/365$ to the cargo owner, where r is the cost of capital of the cargo owner (expressed as an annual interest rate). This loss will be in terms of lost income due to the delayed sale of the cargo. Therefore, it is straightforward to see that $\beta = p \cdot r/365$. We assume that the cargo owner's cost of capital is equal to $r = 5\%$. In the base scenario we also assume an average cargo value of 10.950 $ per ton (this can refer to expensive such as electronics etc.) therefore β is equal to 1.5 \$ per ton cargo per day.

It is obvious that the results depend much on fuel price, charter costs and also the inventory costs. Fuel prices and charter rates are very volatile, therefore a sensitivity analysis is also presented for a selected instance, see Section 5.4.

5.2. Results from different problem variants

As mentioned earlier, by setting the parameters differently we obtain different variations of the problem. Here we take instance G3_4 as an example to examine the solutions of the following four variations:

1. **Min total cost** ($F, U, \alpha, \beta > 0$): this is the general case where the parameters (a) fuel price, (b) state of the market (freight rate), (c) inventory cost of the cargo, and (d) dependency of fuel consumption on payload are taken into consideration in the routing decision at the operational level. The result for the G3_4 instance is depicted in Figure 2. We also provide details of the found solution in Tables 5, 6 and 7, which represent the set of routes for each ship. The visualization shows the routes allocation, while the table give details about the each leg. For each ship result table , the first column show the ports called in the route. For each port call, the second column specified the operations undertaken. This is done using a 3 digit code where the first letter indicate whether the it is a pickup (P) or a delivery (D) operation. The next two values are the origin and destination of the cargo e.g. P45 is the pickup of cargo going from port 4 to port 5, and the corresponding delivery is thus D45. The remaining columns indicate respectively the next sailing leg, the payload, the speed the travel distance and the sailing time. As it can be seen, in this example, all vessels are deployed and the sailing speeds are the maximum ones in almost all legs.

2. **Min total cost with zero port cargo inventory cost** ($\alpha = 0$ and $F, U, \beta > 0$): the case $\alpha = 0$ assumes that cargo is available at the loading

port in a just-in-time fashion and related waiting or delay costs are zero. In this instance, the small and the large vessels are deployed and the sailing speeds are the maximum ones in almost all legs. Solution details can be found in Appendix in Figure A.4.

3. **Min emission** ($F = \alpha = \beta = 0$ and $U > 0$): the objective in this case is to minimize fuel consumption, which finds the routes and the speeds that consume the minimum amount of fuel. In case the ship wants to minimize total emissions (or equivalently minimize total fuel consumed or total fuel cost), it is straightforward to see that all legs should be sailed at minimum speed. The solution uses only the smallest vessel and the sailing speed in all legs is equal to the minimum speed as expected. Solution details can be found in Appendix in Figure A.5.

4. **Min total trip time** ($U = \alpha = \beta = 0$ and $F > 0$): the problem becomes the minimum total trip time problem, which finds the minimum total duration of all the routes. In this case, the ship will take the maximum speed. The solution shows that only one vessel is used (the largest one) and that the legs are sailed as expected at the highest speed in order to minimize the total time and, thus, the chartering cost. Solution details can be found in Appendix in Figure A.6.

port stop	Pickup/delivery operations	Next leg	payload on the leg (Ktons)	remaining weight to pickup (Ktons)	speed (knots)	Distance (nautical miles)	sailing time (days)
0		0–4	0	23	13	0	0
4	P45	4–5	7	16	13	512	1.641
5	D45 P53	5–3	7	9	13	1174	3.763
3	D53 P31	3–1	9	0	13	701	2.247
1	D31	1–0	0	0	13	472	1.513

Table 5: Detailed solution for ship 1 of instance G3_4.

It is important to realize that different objective functions will generally produce very different solutions to the same instance, as it has be shown in the previous examples. In the last two cases the results are as expected and in line with [13]. In the first two cases and especially in the general one (cost

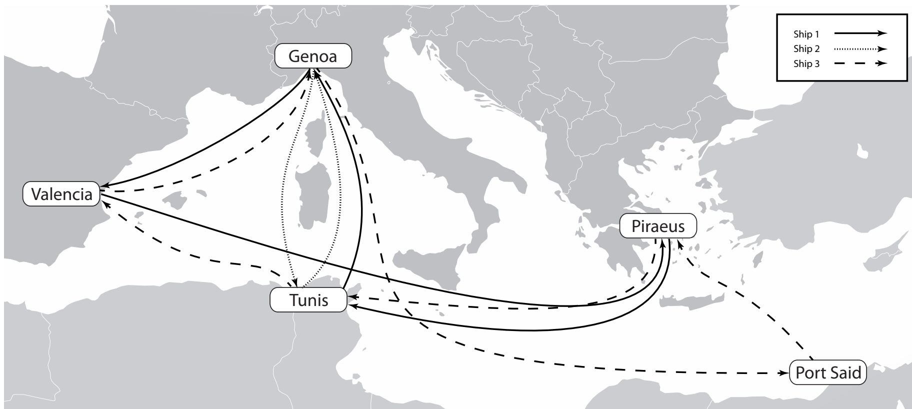


Figure 2: Solution with minimum cost for instance G3_4.

port stop	Pickup/delivery operations	Next leg	payload on the leg (Ktons)	remaining weight to pickup (Ktons)	speed (knots)	Distance (nautical miles)	sailing time (days)
0		0–4	0	14	14	0	0
4	P41	4–1	5	9	14	472	1.405
1	D41 P14	1–4	9	0	14	472	1.405
4	D14	4–0	0	0	13.719	0	0

Table 6: Detailed solution for ship 2 of instance G3_4.

port stop	Pickup/delivery operations	Next leg	payload on the leg (Ktons)	remaining weight to pickup (Ktons)	speed (knots)	Distance (nautical miles)	sailing time (days)
0		0–4	0	17	16	0	0
4	P42	4–2	1	16	16	1446	3.766
2	P23 D42 P25 P21	2–3	15	1	16	619	1.612
3	D23	3–1	14	1	16	701	1.826
1	P15 D21	1–5	6	0	16	560	1.458
5	D15 D25	5–0	0	0	15.968	512	1.336

Table 7: Detailed solution for ship 3 of instance G3_4.

minimization) the results depend on the parameters of the problem. To give a better overview we present, in Table 8, the solutions to all four variants. For each variant, the total sailing distance, the total sailing time, the total cost, the total amount of fuel consumed, the total chartering cost, the total port inventory cost and the total in-transit inventory cost over all the routes in the solution are given.

As we can see in Table 8, in the minimum total trip time scenario the large ship is only deployed and sails the minimum total distance at the maximum speed, thus, the total sailing time is the least one (15.5 days) under this scenario. The reason this ship is chosen is that its maximum speed is the highest, among all ship types. On the other extreme side, one vessel is used again under the minimum emissions scenario sailing at the slowest speed for a total of 64.6 days. This is the smallest ship which has the lowest, among all ships, fuel consumption, and the solution would have that ship alone serve all cargoes using as much time as it would take.

In the quest for environmentally optimal solutions, one might actually assume that if the minimum distance route is sailed at the minimum possible speed in all legs, this would minimize emissions. However, it turns out that this is not necessarily the case as the fuel consumption also depends on the payload. In this instance, the solution that gives the minimum emissions actually has a total distance traveled that is longer than those under the other three objectives.

In the minimum cost scenarios, both when the port inventory cost is zero and in the general case, it seems that the sailing speeds are high due to the high inventory costs.

	min total trip time	min emission	min total cost (JIT)	min total cost
	$U = \alpha = \beta = 0$	$F = \alpha = \beta = 0$	$\alpha = 0$	
Total dist (nautical miles)	5971.0	9299.0	6915.0	7641.0
Total trip time (days)	15.5	64.6	19.7	22.0
Total cost(k$)	165.6	28.5	531.0	759.2
Fuel consumption (tons)	593.8	95.1	487.3	515.9
Fuel cost (k$)	–	28.5	146.2	154.8
Chartering cost(k$)	165.6	–	173.9	189.8
Port inv. cost(k$)	–	–	–	204.7
In-transit inv. cost(k$)	–	–	210.9	210.0
# used ships	1	1	2	3
B&P time (sec)	0.2	0.4	0.5	0.3

Table 8: Results from different problem variants for instance G3_4

5.3. Results of the H-B&P and the CPM

A comparison of the solutions provided by the H-B&P and the CPM are provided in Table 9. For the H-B&P , the total cost as well as the four cost elements are given in columns 2–6. The number of ships used in the solutions and the computational times of the H-B&P are also given in the table. For the CPM , we present the best solution found within 10 hours. The solutions that are proven to be optimal by the CPM are indicated by *. As it can be seen from the table, the H-B&P finds the optimal solution for the first five instances. For the remaining instances, for which the optimal solution is unknown, the solution found by the H-B&P within 30 minutes is much better than the one found by the CPM model. For most of the instances, the H-B&P stops before reaching the time limit, which means the algorithm finishes exploring the branching tree using the heuristic column generation.

5.4. Sensitivity Analysis

To investigate how the fuel price, charter rate and inventory cost affect the solution, we have tested instance G3_4 with different inputs of these parameters. The solution values over these instances are given in Table 10–Table 12. Table 10 provides the results when the fuel price varies from 100 $ per ton to 1300 $ per ton. Table 11 and 12 shows the corresponding results when the relative changes of charter rate are from -60% to +60% and the inventory cost from 0 $ per ton per day to 3 $ per ton per day. With an interest rate of 5% these figures correspond to an average cargo value of 0 to 21.900 $ per ton.

Figure 3 summarizes the results graphically, where the results for average speed, fule consumption and travel distance are plotted. The data is normalized in percentage deviation from the base value; that is 300 $ for fuel price, 0% for the charter rate, and 0.3 $ for the inventory cost. As it can be seen from the results in all cases except when the port cargo inventory cost is low (α equal to 0 or 0.3) the total distance sailed is the same and all ships are being used. In addition, when the fuel price increases, the ships would try to reduce the fuel consumption by taking shorter routes and sailing at a lower speed revealed from

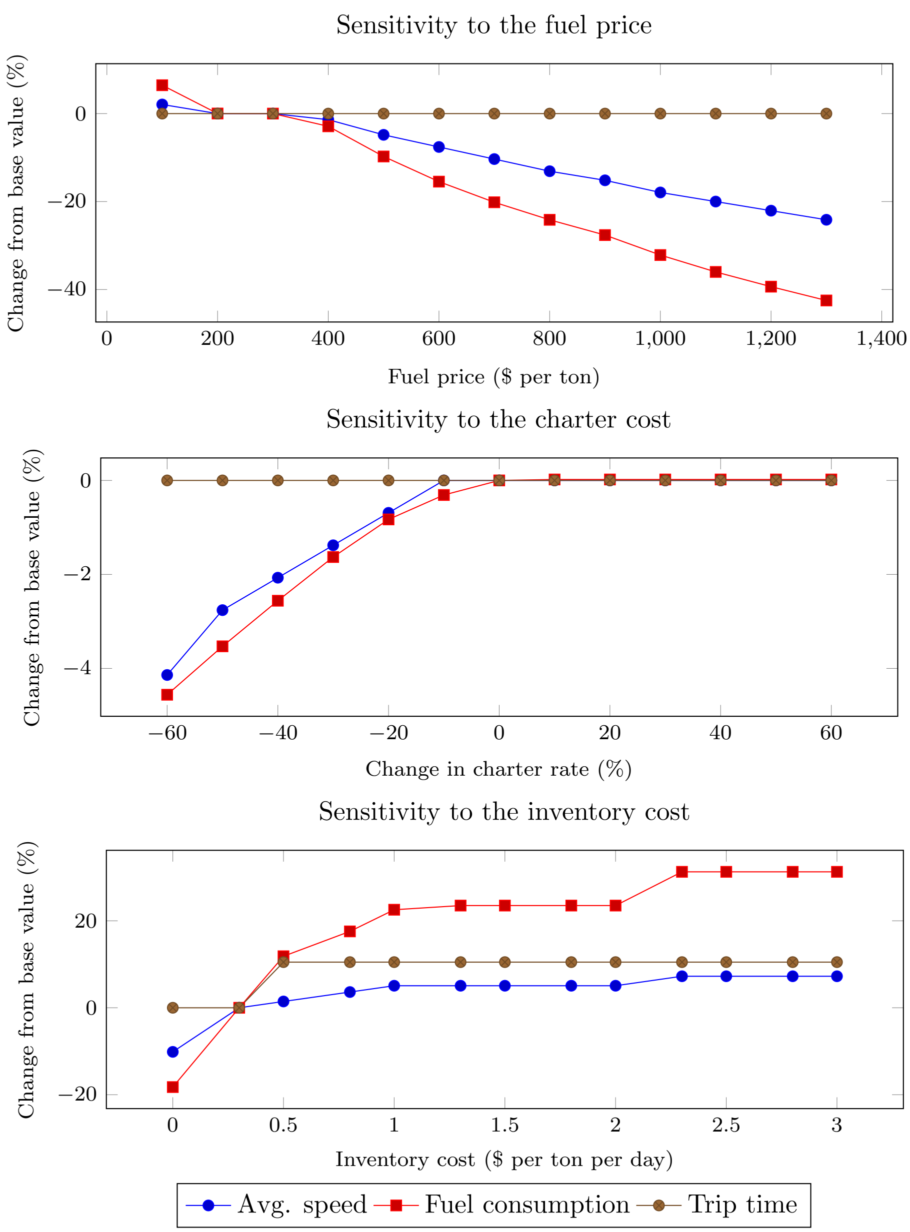


Figure 3: Sensitivity analysis

	H-B&P							CPM
	Fuel cost (K$)	Chartering cost(K$)	Port inv. cost(K$)	In-transit inv. cost(K$)	Total cost (K$)	# of used ships	Computational time (sec)	Total cost (K$)
G1_1	99.5	95.9	176.7	135.9	507.9	1	0.0	507.9*
G1_2	115.9	150.5	153.8	145.6	565.8	2	0.1	565.8*
G1_3	112.6	133.1	108.6	145.6	499.9	2	0.1	499.9*
G1_4	75.8	83.2	93.1	102.1	354.2	1	0.0	354.2*
G1_5	111.1	152.8	130.2	110.3	504.4	3	0.0	504.3*
G2_1	150.6	160.2	262.5	215.1	788.4	3	0.9	1,341.60
G2_2	184.0	192.3	261.1	270.1	907.5	3	0.7	1,340.90
G2_3	163.2	188.1	280.3	227.6	859.3	3	0.7	1,228.90
G2_4	123.7	119.7	168.2	181.3	592.9	2	0.9	947.50
G2_5	127.5	144.0	154.3	182.1	607.9	2	0.9	1,104.60
G3_1	140.5	181.5	133.6	190.1	645.8	3	0.3	798.10
G3_2	118.6	168.5	131.7	145.3	564.1	3	0.6	631.00
G3_3	170.2	214.9	158.4	213.2	756.8	3	0.4	828.20
G3_4	154.8	189.8	204.7	210.0	759.2	3	0.3	863.60
G3_5	172.7	219.5	277.7	225.8	895.8	3	0.3	896.20
G4_1	247.6	249.2	356.0	383.9	1,236.6	3	13.3	7,144.10
G4_2	277.4	275.7	606.1	451.7	1,610.9	3	48.9	7,728.00
G4_3	258.3	263.7	434.9	395.5	1,352.3	3	10.2	7,395.10
G4_4	265.8	284.9	543.3	397.4	1,491.3	3	36.6	7,087.00
G4_5	353.6	386.0	862.1	532.9	2,134.5	3	84.1	8,446.80
G5_1	194.9	230.5	275.8	240.6	941.7	3	5.5	2,140.50
G5_2	156.7	193.3	238.4	184.1	772.5	3	3.2	2,400.90
G5_3	193.9	237.6	262.5	271.4	965.4	3	3.2	3,010.80
G5_4	231.0	265.4	420.9	305.5	1,222.7	3	14.4	2,558.90
G5_5	191.5	225.0	326.0	258.9	1,001.3	3	2.8	3,512.50
G6_1	364.9	387.7	1,126.1	563.8	2,442.5	3	1,800.7	20,523.80
G6_2	291.2	301.5	656.4	448.7	1,697.8	3	1,800.7	15,597.90
G6_3	377.9	393.7	1,032.2	596.7	2,400.5	3	880.5	18,912.70
G6_4	354.6	355.1	954.2	568.5	2,232.3	3	603.6	19,347.30
G6_5	394.5	424.6	1,215.1	587.5	2,621.8	3	1,800.2	20,216.10
G7_1	319.1	354.7	728.6	493.4	1,895.7	3	153.5	9,672.40
G7_2	256.0	294.1	441.5	350.6	1,342.1	3	755.3	7,647.10
G7_3	274.3	332.5	585.1	380.6	1,572.5	3	103.5	5,989.00
G7_4	279.8	283.4	528.0	438.5	1,529.6	3	13.4	8,009.30
G7_5	348.7	402.0	787.5	492.8	2,031.1	3	80.3	9,200.10
G8_1	441.9	479.8	1,447.5	663.4	3,032.5	3	1,721.3	19,592.40
G8_2	435.4	467.3	1,274.9	615.9	2,793.5	3	1,801.5	21,203.50
G8_3	410.3	442.9	1,292.5	621.3	2,767.1	3	1,802.2	20,413.70
G8_4	400.5	423.0	1,248.6	596.1	2,668.2	3	1,800.9	19,972.30
G8_5	393.2	432.2	1,160.9	574.5	2,560.7	3	1,801.8	19,900.90
Average	243.3	269.5	537.5	352.9	1403.2	2.8	428.7	7,500.9

Table 9: Results of the H-B&P and the CPM

Fuel Price ($/ton)	100	200	300	400	500	600	700	800	900	1000	1100.0	1200.0	1300.0
Total dist (nautical miles)	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0
Total trip time (days)	21.5	22.0	22.0	22.3	23.0	23.8	24.5	25.2	25.9	26.7	27.5	28.3	29.1
Total cost(K$)	653.7	707.6	759.2	810.3	858.5	903.5	945.9	986.0	1024.3	1060.4	1094.4	1126.6	1157.0
Fuel consumption (tons)	549.2	516.0	515.9	501.0	465.6	436.1	411.9	391.3	373.4	350.0	330.2	312.9	296.7
Fuel cost (K$)	54.9	103.2	154.8	200.4	232.8	261.7	288.3	313.0	336.1	350.0	363.3	375.5	385.7
Chartering cost(K$)	193.1	189.8	189.8	193.0	200.0	206.9	213.4	219.7	225.6	233.3	240.5	247.3	254.0
Port inv. cost(K$)	199.7	204.7	204.7	204.8	205.2	205.5	206.2	207.4	209.0	215.0	220.7	226.3	232.4
In-transit inv. cost(K$)	206.0	210.0	210.0	212.2	220.6	229.5	238.0	246.0	253.6	262.1	270.0	277.5	285.0
# used ships	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0
Average speed (knot)	14.8	14.5	14.5	14.3	13.8	13.4	13.0	12.6	12.3	11.9	11.6	11.3	11.0
B&P time (sec)	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4

Table 10: Sensitivity to the fuel price

Relative change of freight rate	-60%	-50%	-40%	-30%	-20%	-10%	0%	+10%	+20%	+30%	+40%	+50%	+60%
Total dist (nautical miles)	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0
Total trip time (days)	23.0	22.7	22.4	22.2	22.1	22.0	22.0	22.0	22.0	22.0	22.0	22.0	22.0
Total cost(K$)	643.4	663.1	682.6	702.0	721.1	740.2	759.2	778.2	797.1	816.1	835.1	854.1	873.0
Fuel consumption (tons)	492.4	497.7	502.7	507.5	511.6	514.3	515.9	516.0	516.0	516.0	516.0	516.0	516.0
Fuel cost (K$)	147.7	149.3	150.8	152.3	153.5	154.3	154.8	154.8	154.8	154.8	154.8	154.8	154.8
Chartering cost(K$)	79.6	98.1	116.4	134.6	152.8	171.3	189.8	208.7	227.7	246.7	265.7	284.6	303.6
Port inv. cost(K$)	204.7	204.7	204.7	204.7	204.7	204.7	204.7	204.7	204.7	204.7	204.7	204.7	204.7
In-transit inv. cost(K$)	211.4	211.0	210.7	210.4	210.1	210.0	210.0	210.0	210.0	210.0	210.0	210.0	210.0
# used ships	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0
Average speed (knot)	13.9	14.1	14.2	14.3	14.4	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5
B&P time (sec)	0.4	0.4	0.4	0.5	0.4	0.4	0.3	0.4	0.4	0.4	0.4	0.4	0.4

Table 11: Sensitivity to the charter cost

$\alpha = \beta$ ($/ton/day)	0.0	0.3	0.5	0.8	1.0	1.3	1.5	1.8	2.0	2.3	2.5	2.8	3.0
Total dist (nautical miles)	6915.0	6915.0	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0	7641.0
Total trip time (days)	23.2	20.9	22.7	22.3	22.0	22.0	22.0	22.0	22.0	21.6	21.6	21.6	21.6
Total cost(k$)	307.3	400.9	480.5	551.3	620.9	690.1	759.2	828.3	897.4	966.4	1034.0	1101.6	1169.2
Fuel consumption (tons)	341.5	417.7	467.2	491.1	511.9	515.9	515.9	515.9	515.9	548.3	548.3	548.3	548.3
Fuel cost (k$)	102.4	125.3	140.2	147.3	153.6	154.8	154.8	154.8	154.8	164.5	164.5	164.5	164.5
Chartering cost(k$)	204.9	186.0	197.4	193.4	190.4	189.8	189.8	189.8	189.8	193.3	193.3	193.3	193.3
Port inv. cost(k$)	0.0	50.9	68.4	102.5	136.5	170.6	204.7	238.8	272.9	299.5	332.8	366.1	399.4
In-transit inv. cost(k$)	0.0	38.7	74.5	108.0	140.5	175.0	210.0	244.9	279.9	309.0	343.3	377.7	412.0
# used ships	2.0	2.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0
Average speed (knot)	12.4	13.8	14.0	14.3	14.5	14.5	14.5	14.5	14.5	14.8	14.8	14.8	14.8
B&P time (sec)	0.3	0.4	0.4	0.4	0.4	0.4	0.3	0.3	0.4	0.3	0.5	0.4	0.3

Table 12: Sensitivity to the inventory cost

the increasing trip time. The increase in freight rate does not seem to affect the speeds that much as the average speed remains the same in most of the cases. Finally, the figure shows that increases in the inventory cost parameters ($\alpha = \beta$) lead to higher average speeds in order to reduce the trip time and thus the total inventory costs.

6. Conclusions

This paper has developed models that optimize ship speed for a spectrum of routing scenarios and for several variants that concern the objective function to be optimized. The paper extends the work presented in [13] to the multiple ship case and contributes to further research in this area, for instance in multiple ship problems where many of the properties identified in the single ship case are still valid. To our knowledge, this is the only paper in the maritime OR/MS literature that addresses a multiple ship scenario in which all of (a) the fuel price, (b) the market freight rate, (c) the dependency of fuel consumption on payload and (d) the cargo inventory costs are taken into account. In the quest for a balanced economic and environmental performance of maritime transport, we think that this work can provide useful insights.

References

[1] H. N. Psaraftis, C. A. Kontovas, Speed models for energy-efficient maritime transportation: A taxonomy and survey, Transportation Research Part C: Emerging Technologies 26 (2013) 331–351.

[2] N. A. Papadakis, A. N. Perakis, Deterministic minimal time vessel routing, Operations Research 38 (3) (1990) 426–438.

[3] A. Dec, D. Frangopol, Real-time risk of ship structures integrating structural health monitoring data:application to multi-objective optimal ship routing, Ocean Engineering 96 (2015) 312329.

[4] D. Ronen, The effect of oil price on the optimal speed of ships, Journal of the Operational Research Society (1982) 1035–1040.

[5] A. Perakis, D. Jaramillo, Fleet deployment optimization for liner shipping part 1. background, problem formulation and solution approaches, Maritime Policy and Management 18 (3) (1991) 183–200.

[6] J. Devanney, S. Beach, Solving transport networks in which supply and demand are elastic, Tech. rep., Center for Tankship Excellence (2007). URL `http://c4tx.org/ctx/pub/elnet.pdf`

[7] T. E. Notteboom, B. Vernimmen, The effect of high fuel costs on liner service configuration in container shipping, Journal of Transport Geography 17 (5) (2009) 325–337.

[8] I. Norstad, K. Fagerholt, G. Laporte, Tramp ship routing and scheduling with speed optimization, Transportation Research Part C: Emerging Technologies 19 (5) (2011) 853–865.

[9] P. Cariou, A. Cheaitou, The effectiveness of a european speed limit versus an international bunker-levy to reduce co 2 emissions from container shipping, Transportation Research Part D: Transport and Environment 17 (2) (2012) 116–123.

[10] K. G. Gkonis, H. N. Psaraftis, Modelling tankers: optimal speed and emissions, in: Proceedings SNAME 2012 Annual Meeting, 2012.

[11] L. M. Hvattum, I. Norstad, K. Fagerholt, G. Laporte, Analysis of an exact algorithm for the vessel speed optimization problem, Networks 62 (2) (2013) 132–135.

[12] K. Fagerholt, D. Ronen, Bulk ship routing and scheduling: solving practical problems may provide better results, Maritime Policy & Management 40 (1) (2013) 48–64.

[13] H. N. Psaraftis, C. A. Kontovas, Ship speed optimization: Concepts, models and combined speed-routing scenarios, Transportation Research Part C: Emerging Technologies 44 (2014) 52–69.

[14] M. Doudnikoff, R. Lacoste, Effect of a speed reduction of containerships in response to higher energy costs in sulphur emission control areas, Transportation Research Part D: Transport and Environment 28 (2014) 51–61.

[15] H. Andersson, K. Fagerholt, K. Hobbesland, Integrated maritime fleet deployment and speed optimization: Case study from roro shipping, Computers & Operations Research 55 (2015) 233–240.

[16] K. Fagerholt, N. T. Gausel, J. G. Rakke, H. N. Psaraftis, Maritime routing and speed optimization with emission control areas, Transportation Research Part C: Emerging Technologies 52 (2015) 57–73.

[17] C. Li, X. Qi, C.-Y. Lee, Disruption recovery for a vessel in liner shipping, Transportation Science 49 (4) (2015) 900–921.

[18] J. Xia, K. X. Li, H. Ma, Z. Xu, Joint planning of fleet deployment, speed optimization, and cargo allocation for liner shipping, Transportation Science 49 (4) (2015) 922–938.

[19] Y. Du, Q. Chen, J. S. L. Lam, Y. Xu, J. X. Cao, Modeling the impacts of tides and the virtual arrival policy in berth allocation, Transportation Science 49 (4) (2015) 939–956.

[20] E. F. Magirou, H. N. Psaraftis, T. Bouritas, The economic speed of an oceangoing vessel in a dynamic setting, Transportation Research Part B: Methodological 76 (2015) 48–67.

[21] M. Wen, S. Ropke, H. Petersen, R. Larsen, O. Madsen, Full-shipload tramp ship routing and scheduling with variable speeds, Computers & Operations Research 70 (2016) 1–8.

[22] C. Barnhart, E. Johnson, G.L.Nemhauser, M. Savelsbergh, P. Vance, Branch-and-price: column generation for solving huge integer programs, Operations Research 46 (3) (1998) 316–329.

[23] S. Irnich, Desaulniers, Shortest path problems with resource constraints, Springer US, 2005.

[24] M. Desrochers, J. Desrosiers, M. Solomon, A new optimization algorithm for the vehicle routing problem with time windows, Operations Research 40 (2) (1992) 342–354.

[25] J. Linderoth, M. Savelsbergh, A computational study of search strategies for mixed integer programming, INFORMS Journal Of Computing 11 (2) (1999) 173–187.

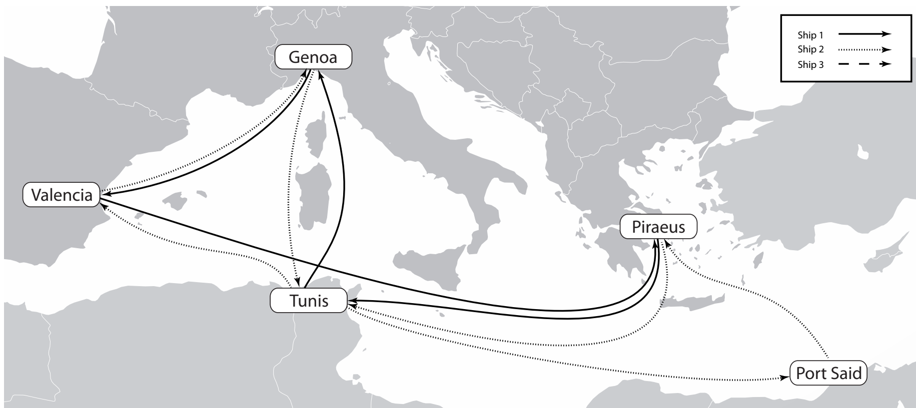
[26] F. Rossi, P. Van Beek, T. Walsh, Handbook of constraint programming, Elsevier, 2006.

[27] S. Dabia, S. Ropke, T. Van Woensel, T. De Kok, Branch and price for the time-dependent vehicle routing problem with time windows, Transportation Science 47 (3) (2013) 380–396.

[28] Gecode, 2015.
URL `http://www.gecode.org`

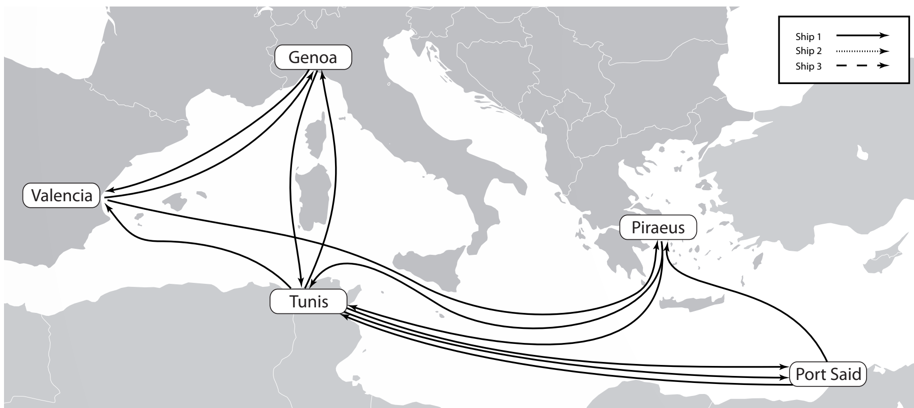
[29] B. D. Brouer, J. F. Alvarez, C. E. Plum, D. Pisinger, M. M. Sigurd, A base integer programming model and benchmark suite for liner-shipping network design, Transportation Science 48 (2) (2013) 281–312.

Appendix A. Results from instance G3_4



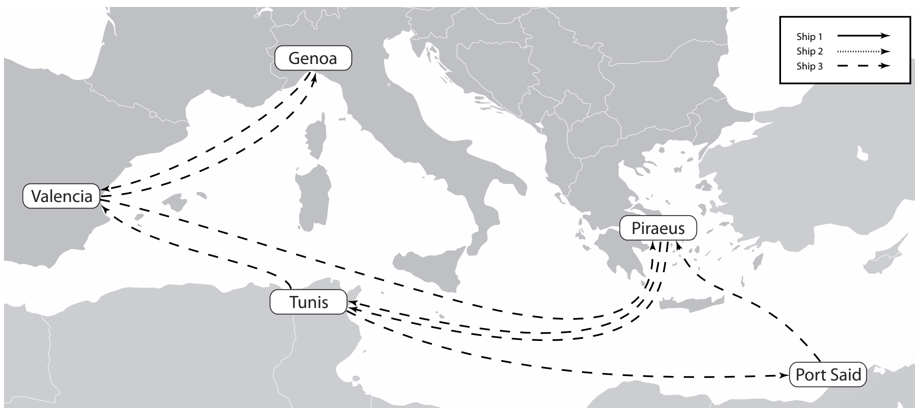
SHIP ID	1						
port	Pickup/delivery	Next	payload on the leg	remaining weight	speed	Distance	sailing time
stop	operations	leg	(Ktons)	to pickup (Kton)	(knots)	(nautical mile)	(days)
0	0–4	0	32	13	0	0	
4	P45	4–5	7	25	13	512	1.641
5	D45 P53	5–3	7	18	13	1174	3.763
3	D53 P31	3–1	9	9	13	701	2.247
1	D31 P14	1–4	9	0	13	472	1.513
4	D14	4–0	0	0	13	0	0
SHIP ID	3						
port	Pickup/delivery	Next	payload on the leg	remaining weight	speed	Distance	sailing time
stop	operations	leg	(Ktons)	to pickup (Ktons)	(knots)	(nautical miles)	(days)
0	0–4	0	22	15.968	0	0	
4	P42 P41	4–1	6	16	16	472	1.229
1	D41	1–2	1	16	16	1192	3.104
2	D42 P23 P25 P21	2–3	15	1	16	619	1.612
3	D23	3–1	14	1	16	701	1.826
1	P15 D21	1–5	6	0	16	560	1.458
5	D15 D25	5–0	0	0	15.968	512	1.336
	s				Total	6915	19.729

Figure A.4: Solution with minimum cost (JIT) for instance G3_4.



SHIP ID	1						
port stop	Pickup/delivery operations	Next leg	payload on the leg (Ktons)	remaining weight to pickup (Ktons)	speed (knots)	Distance (nautical miles)	sailing time (days)
0		0–4	0	54	6	0	0.0
4	P45	4–5	7	47	6	512	3.6
5	D45	5–4	0	47	6	512	3.6
4	P41 P42	4–1	6	41	6	472	3.3
1	D41	1–2	1	41	6	1192	8.3
2	P23 D42 P25	2–3	6	35	6	619	4.3
3	D23	3–1	5	35	6	701	4.9
1	P15	1–5	6	34	6	560	3.9
5	D15 D25 P53	5–3	7	27	6	1174	8.2
3	D53 P31	3–1	9	18	6	701	4.9
1	D31	1–2	0	18	6	1192	8.3
2	P21	2–1	9	9	6	1192	8.3
1	D21 P14	1–4	9	0	6	472	3.3
4	D14	4–0	0	0	6	0	0
					Total	9299	64.576

Figure A.5: Solution with minimum emissions for instance G3_4.



SHIP ID	3						
port stop	Pickup/delivery operations	Next leg	payload on the leg (Ktons)	remaining weight to pickup (Ktons)	speed (knots)	Distance (nautical miles)	sailing time (days)
0		0–4	0.0	54.0	16.0	0	0.0
4	P41 P45 P42	4–5	13.0	41.0	16.0	512	1.3
5	D45 P53	5–3	13.0	34.0	16.0	1174	3.1
3	D53 P31	3–1	15.0	25.0	16.0	701	1.8
1	D41 D31	1–2	1.0	25.0	16.0	1192	3.1
2	P23 D42 P25 P21	2–3	15.0	10.0	16.0	619	1.6
3	D23	3–1	14.0	10.0	16.0	701	1.8
1	P15 D21 P14	1–5	15.0	0.0	16.0	560	1.5
5	D15 D25	5–4	9.0	0.0	16.0	512	1.3
4	D14	4–0	0.0	0.0	16.0	0	0.0
					Total	5971	15.5

Figure A.6: Solution with minimum trip time for instance G3_4.